Model of Transient Aerosol Particle Deposition in Fibrous Media with Dendritic Pattern

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When a suspension of fine solid particles in a gaseous medium flows through a fibrous filter, particles deposit on the fibers forming chainlike agglomerates known as dendrites. This deposition pattern is responsible for the intrinsically transient behavior of the filter, leading to drastic increases of the filtration efficiency and of the pressure drop. Related phenomena are observed when aerosols flow through other types of porous media (for example, granular beds), or next to duct walls, around immersed objects, etc. A theoretical model of the particle dendrite growth was proposed recently by Payatakes and Tien. Here a revised and generalized version of that model is developed. The following major revisions are made: allowance is made for collisions with a particle in a given dendrite layer that lead to retension in the same layer, radial as well as angular contributions to deposition are considered, and the dendrite layer adjacent to the collector is allowed to contain more than one particle. These changes lead to a substantially more realistic theoretical model. Expressions for the transient behavior of a filter of differential thickness are obtained, based entirely on first principles. These, as it has been shown in a previous publication, can be used to predict the dynamic behavior of a macroscopic fibrous filter. The use and behavior of this model is demonstrated in the simple case of deposition by pure interception. The present treatment of deposition by pure interception is more rigorous than and supersedes that adopted in previous works.

SCOPE

In recent years, industry's need for methods to remove fine particulates from gaseous streams efficiently and inexpensively has increased markedly. On the one hand, concern for the protection of the environment, health, and quality of life make the removal of particles, especially those in the submicron range, from gaseous emissions (for instance, coal fired boilers, cement kilns, asphalt plants, etc.) imperative. On the other hand, several new (and old) industrial processes involve one or more stages where removal of fine particles from gaseous streams is required inherently (removal of particles and viruses from air supply in aseptic fermentation, removal of particulate mineral material and char from the product of coal gasifiers, etc.). Currently, there are intense efforts by industrial research and development groups, the EPA, and university research teams to improve existing gas-solid separation methods and/or develop new ones with even better performance characteristics in terms of efficiency, energy consumption, compactness, and temperature tolerance.

Filtration of solid particle-gas suspensions through fibrous filters has a number of important advantages. First, it is highly efficient, even in the range of submicron particles where it matches or outperforms any other known method. Second, it requires relatively low energy consumption per standard unit volume of gas treated. Third, it lends itself to the design of systems which are significantly more compact than competitive ones of equal capacity and comparable efficiency. Its main drawback seems to be the difficulty with which fibrous filters are cleaned. At present, fibrous filters are used in applications where the highest efficiency is demanded and where disposal of the filter elements after complete loading is acceptable (clean rooms, absolutely clean air supply for laboratory applications, emergency filtration systems for radioactive aerosols, absolutely clean air supply for aseptic fermentation processes, etc.). Current efforts by industry to develop effective methods for the repetitive cleaning of fibrous filters are motivated by the many substantial advantages of this particle-gas separation method.

Rational design of particle removal systems based on fibrous filter elements will require quantitative understanding of the transient behavior of the deposition phenomenon. Unfortunately, the overwhelming majority of past theoretical works relates to the deposition in a clean (new) fibrous filter, a condition that lasts only for a negligibly small fraction of the service life of the filter. Only recently has a serious effort been made to predict the transient behavior of fibrous filters from basic principles [Payatakes and Tien (1976), Payatakes (1976*a*, *b*)]. The present work provides a revised and improved approach to the problem.

CONCLUSIONS AND SIGNIFICANCE

A theoretical model is developed which predicts the expected configuration and rate of growth of a single dendrite as a function of angular position on the fiber and of time [Equation (11)]. Analytical expressions describing the transient behavior of a fibrous filter of differential thickness are developed [Equations (18), (20), and (31)], making use of the single dendrite model, of the dendrite age distribution function, and of the dendrite number distribution function. Considerable simplification of these expressions is obtained based on the assumption of an initial period of unhindered dendrite growth, [Equations (35) to (37)]. The transient behavior of a fibrous filter of finite thickness is obtained readily with the integration method given in Payatakes (1976*b*), provided that expressions giving the effect of amount deposited on the local filter coefficient and on the local pressure gradient are available. Such expressions are developed here using the corresponding relations for the filter of differential thickness [Equations (42) and (43)]. The use and behavior of the model are demonstrated for the simple case of particle deposition by interception alone. Other deposition mechanisms can be incorporated without any difficulties of fundamental nature, and this is part of planned future work.

The present work is based on a number of assumptions and involves several simplifications. In spite of this, it does not lose sight of the most important characteristics of dendrite growth and of its effects on filter behavior. It is, therefore, regarded with cautious optimism as suitable framework for the development of a mathematical tool for the prediction of transient filter performance based on first principles. It is expected, further, that it will prove useful in the modeling of the transient behavior of deep-bed filtration of liquid-solid suspensions and other related phenomena.

Future work should extend beyond the period of unhindered growth and also beyond the onset of particle cluster reentrainment.

The process of deposition of aerosol particles in fibrous media is exceedingly complicated. Briefly, one can distinguish at least four different regimes, although this classification is rather arbitrary, and considerable overlapping should be expected. First, when the filter is new, particles deposit directly on the fiber surfaces. This stage is very brief compared to the total loading period, and, as will be seen below, in principle it terminates as soon as it begins. Second, there is a period during which particles deposit not only directly on fibers but also and preferentially on already deposited particles forming particle dendrites. This stage has been observed experimentally and recorded photographically by Leers (1957) and Billings (1966). Third, given sufficient time the dendrites grow to the extent that they intermesh with their neighbors forming a coat of usually nonuniform thickness around each fiber. If the adhesion forces are sufficiently strong, reentrainment of particle clusters will be negligible, and the fourth stage will begin when the particle coatings of neighboring fibers bridge the gaps in between, thus creating a secondary porous matrix within the fibrous filter; this matrix can be described as an internal cake. If, on the other hand, the adhesion forces are not strong enough, the drag force exerted on the dendrites, aided by bombardment from newly depositing or merely colliding particles, will cause significant reentrainment of particle clusters. Reentrainment may begin even during the second stage, if the relative magnitude of drag force over adhesion force is high enough. For a given superficial velocity, reentrainment may occur beyond a critical dendrite size that depends on particle size, fiber size, configuration, and dendrite location, as well as shape and location of the nearest neighbors. A particle covered fiber at this state acts as an agglomerator. If reentrainment is significant, then the fourth stage will not involve the formation of an internal cake but rather a continuous reentrainment of particle clusters and subsequent deposition downstream. In this last case, the local filtration efficiency may display a maximum with amount deposited. Finally, an intermediate type of fourth stage is conceivable, in which a cake is formed, but which is subject to significant attrition by reentrainment.

It is possible to have a situation in which the front layers of a fibrous filter are in the fourth stage, while deeper in the filter deposition is in less advanced stages (even the first stage).

The great majority of theoretical works pertaining to aerosol particle deposition are concerned exclusively with the first stage, namely, deposition on clean fibers (or other surfaces). An excellent review of these works is given by Davies (1973). No genuine theoretical investigation of

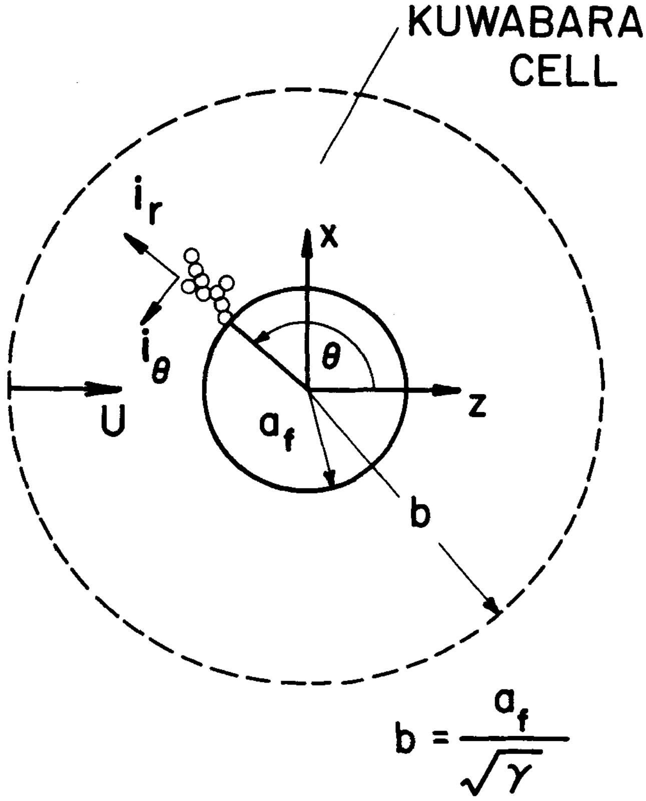

Fig. 1. Kuwabara cell for a fibrous medium. Cartesian and cylindrical coordinates.

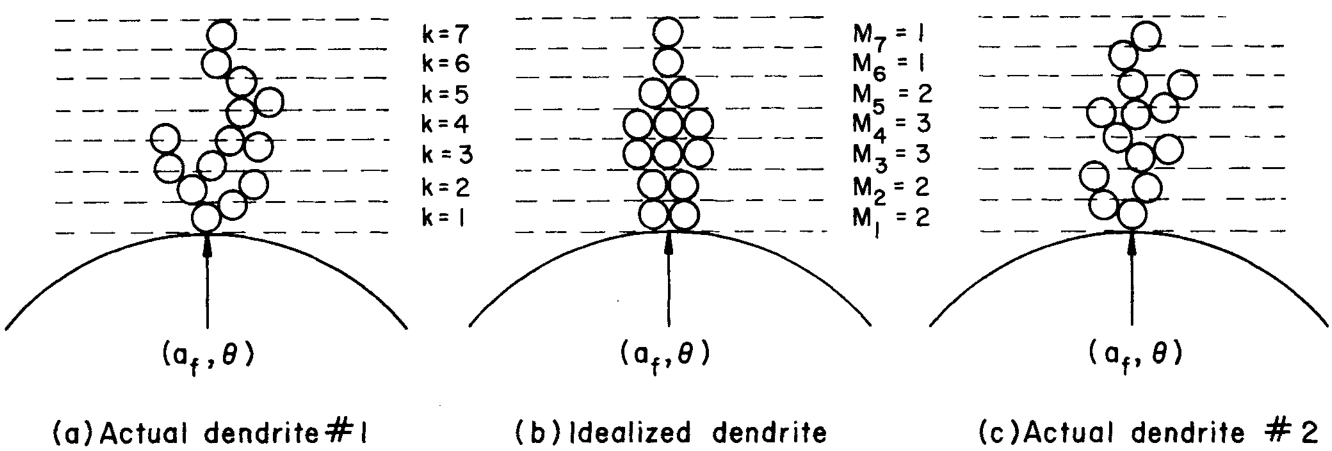

Fig. 2. Idealization of the dendrite configuration (two-dimensional depiction). Different dendrites may correspond to the same idealized configuration; such dendrites are expected to have comparable growth rates and effects on filtration efficiency and pressure drop.

the third and fourth stages has been attempted to date.

Efforts to model the second stage, namely, the regime of particle dendrite formation, have appeared only recently. A theoretical model of single dendrite growth was proposed by Payatakes and Tien (1976) on a preliminary basis and was used further to model the dynamic behavior of a fibrous mat of differential thickness. That work was extended in Payatakes (1976*a*), where analytical expressions giving the dependence of filtration efficiency, amount deposited, and pressure drop on time were derived for the case of pure interception. Payatakes (1976*b*) developed a system of equations describing the transient behavior of macroscopic fibrous filters and used it in conjunction with the results in Payatakes (1976*a*) to predict from basic principles the transient behavior of such filters, assuming pure interception.

The present work is concerned mainly with the improvement of the single dendrite growth model, which forms the cornerstone of the works described above. The preliminary model presented by Payatakes and Tien (1976) is limited by two assumptions, both of which are detrimental to its generality: the dendrite layer adjacent to the collector can contain only one particle at most, and particles colliding with the upper half of a dendrite particle become members of the immediately higher layer, whereas particles that might collide with the lower half are accounted for as colliding with the upper half of a particle in the immediately lower layer. A consequence of the latter assumption is that no allowance is made for collisions with particles of a given layer that lead to deposition in the same layer. Both of these assumptions are relaxed in the present formulation, thus leading to a substantially more general and presumably realistic model. In applying the single dendrite growth model to the case of deposition by pure interception, Payatakes and Tien (1976) and Payatakes (1976*a, b*) considered only contributions from the tangential flow component. This limitation leads to substantial underestimation of the rate of deposition, especially at positions close to the forward stagnation point. In the present work, both angular and radial flow contributions are considered. Detailed comparison of the performance of the improved model version with that of the earlier one is given in a forthcoming publication.

SINGLE DENDRITE GROWTH: IMPROVED FORMULATION

Consider a cylindrical Kuwabara (1959) cell with an approach velocity U, as shown in Figure 1. Let (r, θ) be cylindrical coordinates, with θ measured counterclockwise from the downstream stagnation point.* Consider, now, a dendrite composed of uniform particles of diameter d_p and attached on a point on the surface of the fiber with angular coordinate θ. Payatakes and Tien (1976) introduced a convention for the idealization of the structure of particle dendrites to enable their mathematical description in simple terms, Figure 2. According to this convention, the idealized dendrite structure is described mathematically by a set of integer numbers $\{M_k(t; \theta);\ k = 1, 2, 3, \ldots\}$, where M_k is the number of particles in the k^{th} dendrite layer, and t is the time measured from the instant of deposition of the first particle of the dendrite under consideration. Clearly, $\{M_k;\ k = 1, 2, 3, \ldots\}$ are a set of integer valued coupled random processes, and it is reasonable to argue that they should be studied as such. Wang, Tien, and Barot (1976) made a study of particle dendrite growth using stochastic simulation and reported encouraging results. Undoubtedly, efforts in the same direction should be continued and amplified. On the other hand, such an approach will almost certainly not produce a system of equations of manageable complexity capable of predicting the transient filter behavior. It is expected, rather, that the stochastic simulation method will help develop insight into the dendrite growth process and will serve as a tool for testing alternative hypotheses and as a guide for the development of less time consuming, albeit less rigorous, models. In the present study, the main objective is development of a model of the latter type. To this end, let m_k be the expected value of M_k; m_k, therefore, is not restricted to integer values. The problem is to obtain analytical expressions for $\{m_k(t; \theta);\ k = 1, 2, 3, \ldots\}$. We set

$$\frac{dm_k}{dt} = R^{(\theta)}_{k-1,k} + R^{(\theta)}_{k,k} + R^{(r)}_{k-1,k} + R^{(r)}_{k,k} \quad \text{for} \quad k = 1, 2, 3, \ldots \tag{1}$$

where

$R^{(\theta)}_{k-1,k}$ = rate of increase of m_k by deposition on particles occupying the $(k-1)$st layer due to the angular flow component

$R^{(\theta)}_{k,k}$ = rate of increase of m_k by deposition on particles already occupying the k^{th} layer, due to

* In Payatakes and Tien (1976) and Payatakes (1976*a, b*), θ is measured clockwise from the forward stagnation point. This difference should be kept in mind when results in those works are compared with the corresponding results of the present study.

the angular flow component

$R^{(r)}_{k-1,k}$ = rate of increase of m_k by deposition on particles occupying the $(k-1)$st layer due to the radial flow component

$R^{(r)}_{k,k}$ = rate of increase of m_k by deposition on particles already occupying the k^{th} layer, due to the radial flow component

It is assumed that terms involving addition of particles to the k^{th} layer by deposition onto particles occupying the $(k+1)$st layer are negligibly small [$R^{(\theta)}_{k+1,k} \cong 0$, $R^{(r)}_{k+1,k} \cong 0$]. There are several reasons for this assumption. First, approach to the lower side of a particle in a dendrite is considerably more hindered than it is to the rest of the particle. This is especially true for young dendrites. Second, owing to the velocity distribution around the fiber, the rate of deposition at the upper portions of a particle is higher than at the lower part, even without accounting for steric hindrance. Third, as will be seen below, a fraction of the number of particles that could collide with a particle in the $(k+1)$st layer, owing to the tangential flow component, to become members of the k^{th} layer would also collide with the underlying particle in the k^{th} layer and are accounted for in $R^{(\theta)}_{k,k}$. Finally, most of the forming dendrites grow in the upstream half of the fiber with the only exception of deposition by Brownian motion alone), and in this case $R^{(r)}_{k+1,k}$ is nil. Proceeding with the formulation, we set

$$\left.\begin{aligned} &R^{(\theta)}_{0,1} = 0 \\ &R^{(\theta)}_{k-1,k} = \alpha\phi^{(\theta)}_{k-1,k}\, m_{k-1}\left(1 - \frac{m_k}{\rho m_{k-1}}\right), \\ &\qquad\qquad k = 2, 3, \ldots \end{aligned}\right\} \tag{2}$$

$$R^{(\theta)}_{k,k} = \alpha\phi^{(\theta)}_{k,k}\, m_k, \qquad k = 1, 2, 3, \ldots \tag{3}$$

$$\left.\begin{aligned} &R^{(r)}_{0,1} = 0 \\ &R^{(r)}_{k-1,k} = \alpha\phi^{(r)}_{k-1,k}\, m_{k-1}\left(1 - \frac{m_k}{\rho m_{k-1}}\right), \\ &\qquad\qquad k = 2, 3, \ldots \end{aligned}\right\} \tag{4}$$

$$R^{(r)}_{k,k} = \alpha\phi^{(r)}_{k,k}\, m_k, \qquad k = 1, 2, 3, \ldots \tag{5}$$

where α is defined as the number of particles approaching the cylindrical collector (normal to the direction of flow) per unit length per unit time, given by

$$\alpha = d_f U n \tag{6}$$

where U is the average interstitial velocity, and n is the particle number concentration of the approaching aerosol stream. We define $\phi^{(\theta)}_{k-1,k}$ as the fraction of α expected to collide owing to the angular velocity component with a particle occupying the $(k-1)$st layer of the dendrite to become a member of the k^{th} layer. The terms $\phi^{(\theta)}_{k,k}$, $\phi^{(r)}_{k-1,k}$ and $\phi^{(r)}_{k,k}$ are defined accordingly. Finally, ρ is the coordination number, that is, the maximum number of particles in the $(k+1)$st layer which can be attached *directly* to the same particle of the k^{th} layer. Here it will be assumed that $\rho = 3$. Notice that, unlike the original formulation [Payatakes and Tien (1976)], the restriction $m_{k+1} \leqq \rho m_k$ is not imposed in the present model, since the number of members of the $(k+1)$st layer that are attached to other members of the same layer is not bounded. Equations (1) to (5) combined give

$$\frac{dm_k}{dt} \equiv \dot{m}_k = \alpha a_k m_{k-1} + \alpha b_k m_k \quad k = 1, 2, 3, \ldots \tag{7}$$

with

$$\left.\begin{aligned} &a_1 = 0 \\ &a_k = \phi^{(\theta)}_{k-1,k} + \phi^{(r)}_{k-1,k} \qquad k = 2, 3, \ldots \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} &b_1 = \phi^{(\theta)}_{1,1} + \phi^{(r)}_{1,1} \\ &b_k = \phi^{(\theta)}_{k,k} + \phi^{(r)}_{k,k} - \frac{1}{\rho}\left[\phi^{(\theta)}_{k-1,k} + \phi^{(r)}_{k-1,k}\right], \\ &\qquad\qquad k = 2, 3, \ldots \end{aligned}\right\} \tag{9}$$

Equations (7) are to be integrated with the following initial conditions:

$$m_1 = 1 \text{ and } m_k = 0 \quad \text{for} \quad k = 2, 3, \ldots \quad \text{at} \quad t = 0 \tag{10}$$

If we assume that α is constant, the solution is

$$m_k = \sum_{i=1}^{k} C_{ki}(\theta)\exp\left[\alpha b_i(\theta) t\right] \quad \text{for} \quad k = 1, 2, 3, \ldots \tag{11}$$

where

$$\left.\begin{aligned} &C_{11} = 1, \quad C_{21} = \frac{a_2}{(b_1 - b_2)}, \quad C_{22} = \frac{a_2}{(b_2 - b_1)} \\ &C_{k1} = \prod_{2 \leq j \leq k} \frac{a_j}{(b_1 - b_j)}, \\ &C_{ki} = \frac{a_i}{(b_i - b_1)} \prod_{\substack{2 \leq j \leq k \\ j \neq i}} \frac{a_j}{(b_i - b_j)} \\ &\text{for} \quad k = 3, 4, \ldots; \quad i = 2, 3, \ldots, k \end{aligned}\right\} \tag{12}$$

A simple algorithm for the calculation of $\{C_{ki};\ k = 3, 4, \ldots;\ i = 1, 2, \ldots, k\}$ is given below:

$$\left.\begin{aligned} &C_{ki} = \frac{a_k}{(b_i - b_k)} C_{k-1,i} \\ &\qquad i = 1, \ldots, (k-1) \\ &C_{kk} = \frac{a_k}{(b_k - b_1)} \prod_{j=2}^{k-1} \frac{a_j}{(b_k - b_j)} \end{aligned}\right\} \text{for} \quad k = 3, 4, \ldots \tag{13}$$

C_{ki} are dimensionless functions of θ but not of time t and such that

$$\sum_{i=1}^{k} C_{ki}(\theta) = 0 \qquad k = 2, 3, \ldots \tag{14}$$

The forms of a_k, b_k, and C_{ki} depend on the dominant deposition mechanism(s) (see below).

MODEL OF THE TRANSIENT BEHAVIOR OF A FIBROUS FILTER OF DIFFERENTIAL THICKNESS

Consider a fibrous filter of differential thickness δz. The inlet and outlet particle number concentrations are n and $n + \delta n$, respectively ($\delta n < 0$); the inlet and outlet static pressure values are P and $P + \delta P$, respectively ($\delta P < 0$). The filter coefficient is defined by

$$\lambda = -\frac{1}{n}\frac{dn}{dz} \quad (15)$$

Thus

$$\lambda \cong -\frac{1}{n}\frac{\delta n}{\delta z} \quad (16)$$

The value of λ for a clean filter λ_o is given by

$$\lambda_o = \frac{2\gamma}{\pi a_f}\eta_o \quad (17)$$

where η_o is the single clean fiber collection efficiency.

Prediction of Filtration Efficiency Increase

Let τ be the time measured from the instant aerosol starts flowing through the filter, whereas time t is the age of a given dendrite measured from the instant of deposition of its first particle. Now, let $N(\tau, \theta)\delta\theta$ be the number of dendrites per unit fiber length between θ and $\theta + \delta\theta$ at time τ (taking into account both sides of the fiber). $N(\tau, \theta)$ is the dendrite number distribution function. Let also $\chi(t; \tau, \theta)\delta t$ be the fraction of the population of dendrites at time τ and at angle θ with age between t and $t + \delta t$. $\chi(t; \tau, \theta)$ is the dendrite age distribution function. In analogy with Payatakes and Tien (1976), we have from a a simple mass balance

$$\frac{\lambda(\tau)}{\lambda_o} = 1 + \frac{1}{\alpha\eta_o}\int_o^\pi \left\{ \sum_{k=1}^{M(\theta)} \left[\int_o^\tau \dot{m}_k(t;\theta)\, \chi(t;\tau,\theta)\,dt \right] \right\} N(\tau,\theta)\,d\theta \quad (18)$$

where $M(\theta)$ is an integer function of θ defined by

$$m_M(\tau_f;\theta) \geqq \epsilon, \quad m_{M+1}(\tau_f;\theta) < \epsilon \quad (19)$$

where τ_f is the duration of the filtration run (during which the present model applies), and ϵ is a positive number, less than one-half, say $\epsilon = 0.05$. In specifying τ_f, care should be taken to restrict the dendrite growth inside the Kuwabara cell, that is, to have $M(\theta)d_p \leqq b - a_f$ for $0 \leqq \theta \leqq \pi$.

Prediction of Pressure Gradient Increase

In analogy with Payatakes and Tien (1976), one can show that

$$\frac{\delta P(\tau)}{\delta P_o} = 1 + \frac{K}{4\pi\mu U_s}\int_o^\pi \left\{ \sum_{k=1}^{M(\theta)} \left[\int_o^\tau m_k(t;\theta)\, \chi(t;\tau,\theta)\,dt \right] F_{pk}(\theta) \right\} N(\tau,\theta)\,d\theta \quad (20)$$

In the above, δP_o is the pressure drop through the clean filter element, which, based on the Kuwabara (1959) model, is given by

$$\frac{\delta P_o}{\delta z} = -\frac{4\gamma\mu U_s}{a_f^2 K} \quad (21)$$

K is the Kuwabara constant, defined as

$$K = -\frac{1}{2}\ln\gamma - \frac{3}{4} + \gamma - \frac{\gamma^2}{4} \quad (22)$$

and F_{pk} is the drag force on a particle in the k^{th} layer of a dendrite. F_{pk} is exerted in the direction of the main flow (parallel to the z axis) and is given by

$$F_{pk} = -\frac{6\pi\mu a_p}{C_s}\{v_\theta[a_f + (2k-1)a_p, \theta]f_k^\theta \sin\theta - v_r[a_f + (2k-1)a_p, \theta]f_k^r \cos\theta\}, \quad k = 1, 2, \ldots \quad (23)$$

where C_s is the Cunningham correction factor given by

$$C_s = 1 + \left(\frac{\lambda'}{a_p}\right)[1.257 + 0.400\exp(-1.10\, a_p/\lambda')] \quad (24)$$

where λ' is the mean free path of the gas. Based on the Kuwabara model

$$v_r = \frac{U}{2K}\left[2\ln\frac{r}{a_f} - 1 + \gamma + \left(1 - \frac{\gamma}{2}\right)\left(\frac{a_f}{r}\right)^2 - \frac{\gamma}{2}\left(\frac{r}{a_f}\right)^2\right]\cos\theta \quad (25)$$

$$v_\theta = -\frac{U}{2K}\left[2\ln\frac{r}{a_f} + 1 + \gamma - \left(1 - \frac{\gamma}{2}\right)\left(\frac{a_f}{r}\right)^2 - \frac{3\gamma}{2}\left(\frac{r}{a_f}\right)^2\right]\sin\theta \quad (26)$$

The terms f_k^θ and f_k^r are correction factors accounting for the presence of the neighboring particles. At present, these correction factors are not available [except for $k = 1$, Gozen and O'Neil (1971)]; their determination would be tedious but does not present difficulties of a fundamental nature. This determination is part of planned future work.

Introducing Equations (25) and (26) into Equation (24), one obtains

$$F_{pk} = \frac{3\pi\mu a_p U}{KC_s}[W_{rk}f_k^r\cos^2\theta + W_{\theta k}f_k^\theta\sin^2\theta], \quad k = 1, 2, \ldots \quad (27)$$

with

$$W_{rk} = 2\ln X_k - 1 + \gamma + \left(1 - \frac{\gamma}{2}\right)X_k^{-2} - \frac{\gamma}{2}X_k^2, \quad k = 1, 2, \ldots \quad (28)$$

$$W_{\theta k} = 2\ln X_k + 1 + \gamma - \left(1 - \frac{\gamma}{2}\right)X_k^{-2} - \frac{3\gamma}{2}X_k^2, \quad k = 1, 2, \ldots \quad (29)$$

$$X_k = 1 + (2k-1)\frac{a_p}{a_f} \quad k = 1, 2, \ldots \quad (30)$$

Prediction of Specific Deposit

The specific deposit σ is defined as the volume of deposited material per unit filter volume. In analogy with Payatakes and Tien (1976), one gets

$$\sigma(\tau) = \frac{4\gamma a_p^3}{3a_f^2}\int_o^\pi \left\{ \sum_{k=1}^{M(\theta)} \left[\int_o^\tau m_k(t;\theta)\, \chi(t;\tau,\theta)\,dt \right] \right\} N(\tau,\theta)\,d\theta \quad (31)$$

Integration of Equations (18), (20), and (31)

Integration of the right-hand sides of these equations requires knowledge of $\chi(t; \tau, \theta)$ and $N(\tau, \theta)$.

Period of Unhindered Dendrite Growth

The concept of the period of unhindered dendrite growth was introduced in Payatakes (1976*a*). This period,

$0 \leq \tau \leq \tau_{ug}$, is defined as the initial length of time during which the growth of each dendrite can be assumed as essentially unhindered by the growth of other dendrites. A rough estimate of τ_{ug} is given in Payatakes (1976*a*). In light of the behavior of the present model (see sample calculations below), that estimate seems rather conservative, and a period twice as long will be adopted here:

$$\tau_{ug} = \frac{\pi}{100 a_p^2 U n \eta_o} \quad (32)$$

The above estimate of τ_{ug} corresponds to the deposition stage when the average distance between particles deposited directly on the fiber equals approximately four particle diameters. Hence, it seems reasonable to assume as a first approximation that during this period the rate of deposition *directly* on the clean fiber is nearly uniform. This implies that the rate of dendrite initiation is also nearly uniform, and

$$\chi(t; \tau, \theta) \cong \frac{1}{\tau} \quad \text{for} \quad 0 \leq \tau \leq \tau_{ug}; \quad 0 \leq \theta \leq \pi \quad (33)$$

During the same period the dendrite number distribution function can be expressed as

$$N(\tau, \theta) \cong \alpha \eta_o \tau \nu(\theta) \quad \text{for} \quad 0 \leq \tau \leq \tau_{ug}; \quad 0 \leq \theta \leq \pi \quad (34)$$

where ν is a dimensionless function of θ alone, and its form depends on the dominant deposition mechanism(s).

Substituting the expressions for m_k, χ, and N given by Equations (11), (33), and (34), respectively, into Equations (18), (20), and (31), one gets

$$\frac{\lambda(\tau)}{\lambda_o} = \int_o^\pi \left[\sum_{k=1}^{M(\theta)} \sum_{i=1}^{k} C_{ki}(\theta) \exp(\alpha b_i \tau) \right] \nu(\theta) d\theta, \quad \text{for} \quad 0 \leq \tau \leq \tau_{ug} \quad (35)$$

$$\frac{\delta P(\tau)}{\delta P_o} = 1 + \frac{3 a_p \eta_o}{4(1-\gamma) C_s} \int_o^\pi \left\{ \sum_{k=1}^{M(\theta)} \sum_{i=1}^{k} \frac{C_{ki}(\theta)}{b_i(\theta)} [\exp(\alpha b_i \tau) - 1] \cdot [W_{rk} f_k^r \cos^2\theta + W_{\theta k} f_k^\theta \sin^2\theta] \right\} \nu(\theta) d\theta, \quad \text{for} \quad 0 \leq \tau \leq \tau_{ug} \quad (36)$$

$$\sigma(\tau) = \frac{4 \gamma a_p^3 \eta_o}{3 a_f^2} \int_o^\pi \left\{ \sum_{k=1}^{M(\theta)} \sum_{i=1}^{k} \frac{C_{ki}(\theta)}{b_i(\theta)} [\exp(\alpha b_i \tau) - 1] \right\} \nu(\theta) d\theta, \quad \text{for} \quad 0 \leq \tau \leq \tau_{ug} \quad (37)$$

Integration of the right-hand sides of Equations (35) to (37) requires knowledge of $\{a_k(\theta), b_k(\theta); k = 1, 2, \ldots, M(\theta)\}$ and $\nu(\theta)$. These functions depend on the dominant deposition mechanism(s).

Later in the present work, these functions are derived for the case of deposition by interception alone.

TRANSIENT BEHAVIOR OF A FIBROUS FILTER OF FINITE THICKNESS

Payatakes (1976*b*) proposed a system of differential equations akin to those developed by Herzig, Leclerc, and LeGoff (1970), which describes the transient behavior of fibrous filters. Integration of these equations is performed easily if two functions f_λ and f_P, which account for the effect of deposited matter on the local rate of deposition and on the local pressure gradient, respectively, are available. These functions are defined by

$$\frac{\lambda}{\lambda_o} \equiv f_\lambda [\sigma(z, \tau); \boldsymbol{\beta}] \quad (38)$$

and

$$\left(\frac{\partial P}{\partial z} \right)_\tau \Bigg/ \left(\frac{dP}{dz} \right)_{\sigma=0} \equiv f_P [\sigma(z, \tau); \boldsymbol{\delta}] \quad (39)$$

where $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ are parameter vectors. These two functions can be determined empirically by fitting experimental data, treating $\boldsymbol{\beta}$ and $\boldsymbol{\delta}$ as adjustable parameter vectors. This approach is recommended for the design and optimization of filtration systems based on laboratory data, since it removes the need for wasteful trial-and-error procedures. There are, however, great incentives for the development of theoretical expressions for f_λ and f_P. To this end Payatakes (1976*b*) argued that even though expressions of the type of Equations (35) to (37) are derived based on constant feed conditions, they can be used to obtain f_λ and f_P as follows. Let Equation (37) be written as

$$\sigma = F(\alpha \tau) \quad (40)$$

Inverting Equation (40) formally, one gets

$$\alpha \tau = F^{-1}(\sigma) \equiv \Phi(\sigma) \quad (41)$$

Substituting $\Phi(\sigma)$ for $\alpha\tau$ in Equations (35) and (36) results in two expressions for λ/λ_o and $\delta P/\delta P_o$ as functions of σ, in which the concentration n does not appear explicitly or implicitly. Hence, arguing as in Payatakes (1976*b*), we set

$$f_\lambda = \int_o^\pi \left\{ \sum_{k=1}^{M(\theta)} \sum_{i=1}^{k} C_{ki}(\theta) \exp[b_i \Phi(\sigma)] \right\} \nu(\theta) d\theta, \quad \text{for} \quad 0 \leq \tau \leq \tau_{ug} \quad (42)$$

$$f_P = 1 + \frac{3 a_p \eta_o}{4(1-\gamma) C_s} \int_o^\pi \left\{ \sum_{k=1}^{M(\theta)} \sum_{i=1}^{k} \frac{C_{ki}(\theta)}{b_i(\theta)} [\exp[b_i \Phi(\sigma)] - 1] \cdot [W_{rk} f_k^r \cos^2\theta + W_{\theta k} f_k^\theta \sin^2\theta] \right\} \nu(\theta) d\theta, \quad \text{for} \quad 0 \leq \tau \leq \tau_{ug} \quad (43)$$

In practice, f_λ and f_P can be obtained readily as functions of σ by regression or interpolation methods from Equations (35) to (37). This procedure yields simple and accurate approximations to Equations (42) and (43). Details for the use of f_λ and f_P to determine the transient behavior of a fibrous filter can be found in Payatakes (1976*b*).

APPLICATION TO THE CASE OF DEPOSITION BY PURE INTERCEPTION

The present derivation is more rigorous and general than that given in Payatakes and Tien (1976) [and also used in Payatakes (1976*a*, *b*)] and should be considered as superseding it. It is assumed that a given suspended particle follows a streamline of the unperturbed Kuwabara flow field until it approaches a dendrite particle, from

TABLE 1. PARAMETER VALUES USED FOR SAMPLE CALCULATIONS

$d_f = 10\ \mu m$

$\gamma = 0.03$

$d_p = 2\ \mu m$
$n = 1.0\ mm^{-3}$
$\mu = 1.813 \times 10^{-5}\ Pa \cdot s$
$U_s = 150\ mm/s$
$\lambda' = 0.083\ \mu m$
$\rho = 3$
$\eta_o = 3.31 \times 10^{-2}$, [Equation (44)]

$\lambda_o = 0.126\ mm^{-1}$ [Equation (17)]
$\left(\frac{dP}{dz}\right)_o = 12.64\ Pa/mm$ [Equation (21)]

$\tau_{ug} = 6\,129\ s = 102.15$ min [Equation (32)]
$L = 10$ mm
$\epsilon = 0.5$
$\tau_f = 2\tau_{ug}$
$(f_k^r = f_k^\theta = 1)$

which point on it attempts to evade collision with the latter following the corresponding streamline of the flow around it.

In order to account for this effect, the following assumptions are made. In calculating the tangential flow contributions to deposition, a Stokes flow is considered around the target particle; the corresponding approach velocity is taken as equal to the angular velocity component of the unperturbed Kuwabara field at the position of the center of the target particle. In calculating the radial contributions, a similar method is used; a Stokes flow is considered with approach velocity, in this case, equal to that of the radial velocity component of the unperturbed Kuwabara field at a point which is obtained from the position of the center of the target particle by increasing the radial coordinate by one particle diameter. The results are summarized below:

$$\eta_o = \frac{(a_p + a_f)}{2Ka_f} W_{r1} \tag{44}$$

$$\left.\begin{aligned} \nu(\theta) &= -\cos\theta, \quad \text{for} \quad 0 \leq \tau \leq \tau_{ug}; \quad \frac{\pi}{2} \leq \theta \leq \pi \\ \nu(\theta) &= 0 \quad , \quad \text{for} \quad 0 \leq \tau \quad ; \quad 0 \leq \theta < \frac{\pi}{2} \end{aligned}\right\} \tag{45}$$

$$a_k = \Theta_{k-1,k} \sin\theta - \mathcal{R}_{k-1,k} \cos\theta \quad k = 1, 2, \ldots \tag{46}$$

$$b_k = \left(\Theta_{k,k} - \frac{1}{\rho}\Theta_{k-1,k}\right)\sin\theta - \left(\mathcal{R}_{k,k} - \frac{1}{\rho}\mathcal{R}_{k-1,k}\right)\cos\theta \quad k = 1, 2, \ldots \tag{47}$$

with

$$\left.\begin{aligned} &\Theta_{0,1} \equiv 0, \quad \Theta_{1,1} = \frac{0.301 a_p^2}{Ka_f} W_{\theta 1} \\ &\Theta_{k-1,k} = \frac{0.191 a_p^2}{Ka_f} W_{\theta,k-1}, \quad \Theta_{k,k} = \frac{0.601 a_p^2}{Ka_f} W_{\theta k}, \\ &\qquad \text{for} \quad k = 2, 3, \ldots \end{aligned}\right\} \tag{48}$$

$$\left.\begin{aligned} &\mathcal{R}_{0,1} \equiv 0 \\ &\mathcal{R}_{k-1,k} = \frac{15\pi a_p^2}{64Ka_f} W_{rk}, \quad \text{for} \quad k = 2, 3, \ldots; \quad \frac{\pi}{2} \leq \theta \leq \pi \\ &\mathcal{R}_{k,k} = \frac{5\pi a_p^2}{64Ka_f} W_{r,k+1}, \quad \text{for} \quad k = 1, 2, \ldots; \quad \frac{\pi}{2} \leq \theta \leq \pi \end{aligned}\right\} \tag{49}$$

Note that for $0 \leq \theta \leq \pi/2$, no dendrites grow due to interception alone [Equation (45)]. The corresponding expressions for C_{ki} are obtained readily by substituting the above expressions for a_k and b_k into Equation (12). In practice, it is preferable to use the algorithm given by Equation (13) for the calculation of $\{C_{ki};\ k = 3, 4, \ldots;\ i = 1, 2, \ldots, k\}$.

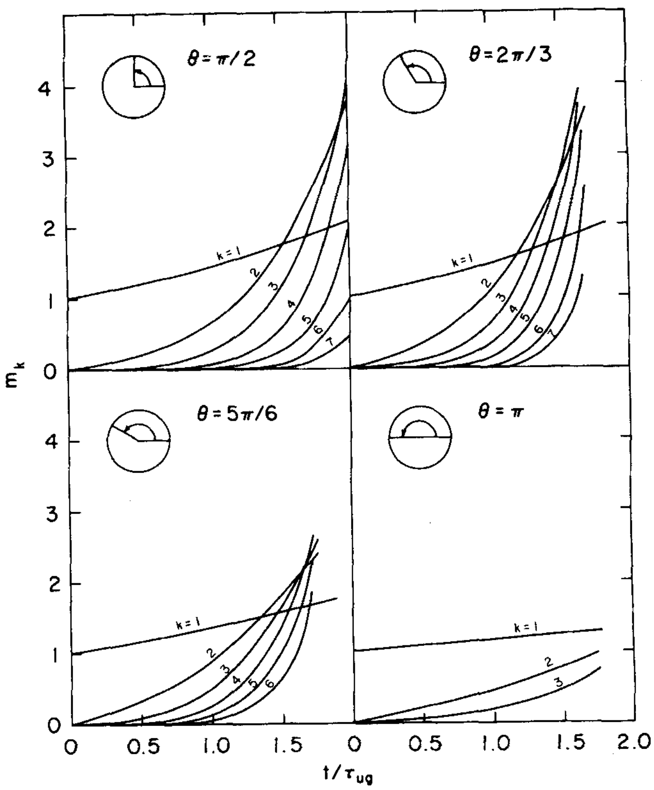

Fig. 3. Calculated profiles of m_k for the conditions given in Table 1.

Calculation of the integrals in the right-hand sides of Equations (35) to (37) is performed routinely with standard numerical methods (quadrature, Simpson's rule, etc.).

SAMPLE CALCULATIONS

The system defined by the parameter values in Table 1 is chosen to demonstrate the use and performance of the present model, assuming deposition by interception alone. The calculated values of m_k are shown in Figure 3 as functions of dendrite age t for four different angles: $\theta = \pi/2, 2\pi/3, 5\pi/6, \pi$. As can be seen, $\{m_k;\ k = 1, 2, \ldots\}$ depend strongly on the angular position. Very vigorous dendrite growth is expected at angles around $2\pi/3$, whereas the expected dendrite size decreases gradually as θ approaches $\pi/2$ or π. This is shown even more clearly in Figure 4, where the expected dendrite configuration is depicted as a function of age and angular position using the two-dimensional idealized representation. These figures are obtained by rounding the corresponding values of m_k to the nearest integer numbers. A few observations can be

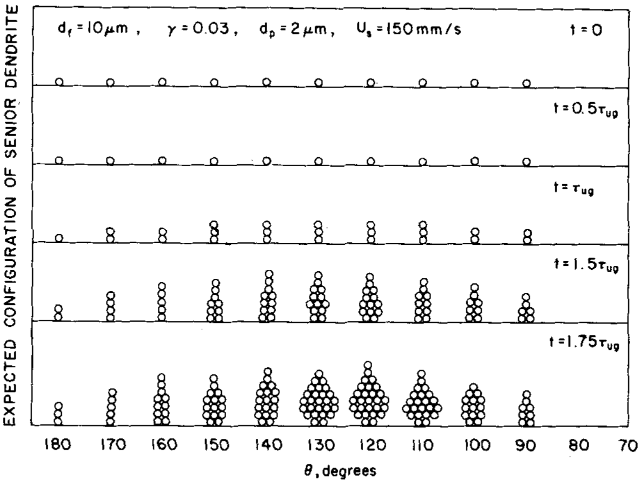

Fig. 4. Two-dimensional idealized representation of the expected dendrite configuration as a function of age and angular position for the conditions given in Table 1.

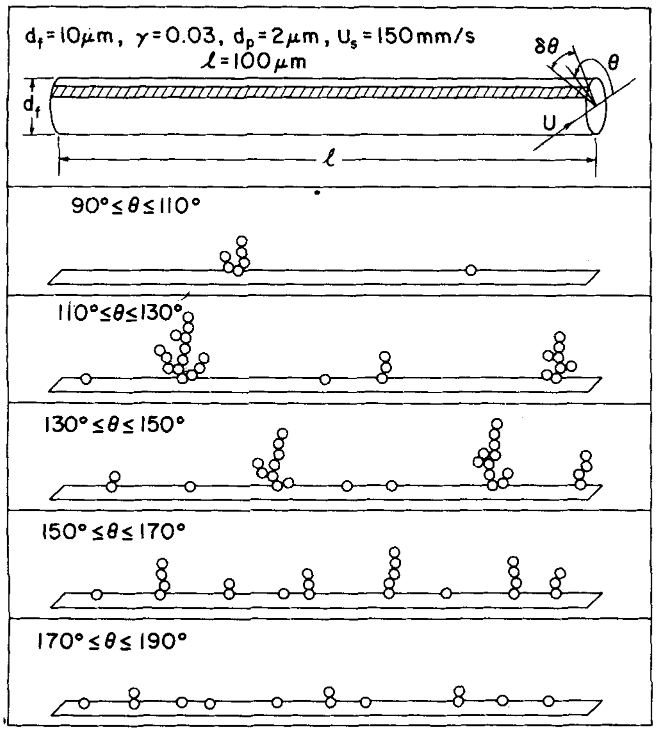

Fig. 6. Typical dendrite deposition corresponding to the expected one at time $\tau = 3\tau_{ug}/2$ for the conditions in Table 1 and $l = 100$ μm. To scale.

made on the basis of these idealized dendrite structures. First, as already mentioned, the rate of growth of a given dendrite displays a maximum at some angle around $2\pi/3$. Second, the minimum growth rate is at the forward stagnation point $\theta = \pi$. It should be emphasized that this remark applies to the rate of growth of a given dendrite, once its first particle has been deposited. On the other hand, according to Equation (45), the rate of deposition of particles directly on the fiber, and therefore the number of dendrites of any size per unit area, is maximum at the forward stagnation point $\theta = \pi$ and decreases monotonically to nil as θ approaches $\pi/2$. Third, dendrites at the forward stagnation point are very lean; dendrites at $\theta = \pi/2$ are also relatively small but are markedly bulkier than those at $\theta = \pi$. Note that the two-dimensional idealized representation used in Figure 4 is somewhat deceiving in that it depicts dendrites considerably

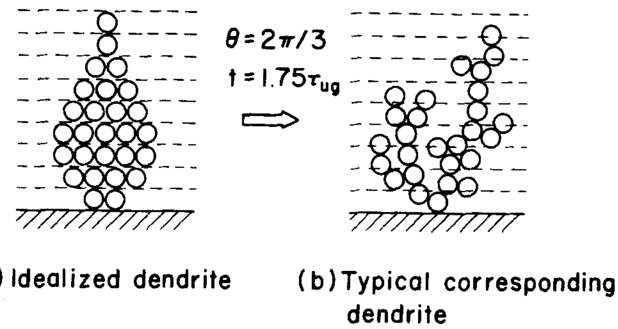

Fig. 5. Typical dendrite of age $t = 1.75\ \tau_{ug}$ corresponding to the expected idealized configuration at $\theta = 2\pi/3$ for the conditions in Table 1.

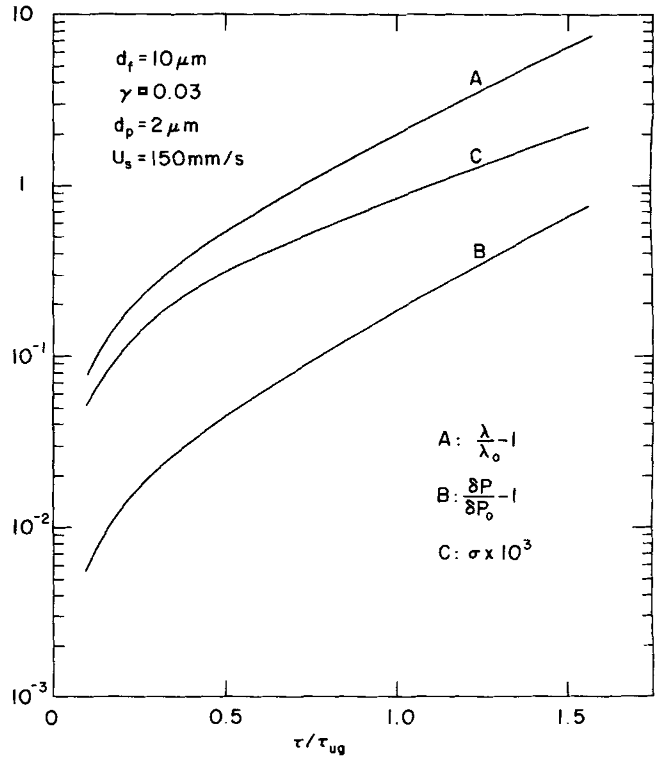

Fig. 7. Transient behavior of a fibrous filter of differential thickness for the conditions in Table 1.

more obese than they are in reality. This is demonstrated in Figure 5, where it is seen that even the bulkiest dendrite, namely, that of age $t = 1.75\ \tau_{ug}$ at $\theta = 2\pi/3$, may represent actual dendrites which contain the same number of particles in corresponding layers but which are considerably leaner owing to branching. This is an important observation confirming in retrospect the validity of several of the assumptions incorporated in the model. Fourth, the rate of new particle additions is very uneven. Initially, very few new particles are expected to deposit on a given incipient dendrite. However, the rate of new additions accelerates rapidly, creating a phenomenon that could be described aptly as a population explosion. This population explosion will ultimately lead to intermeshing of the dendrite with its neighbors, or to a catastrophic event, namely, dendrite reentrainment. Finally, compared to the dendrite configurations predicted with the preliminary model version in Payatakes and Tien (1976), the dendrite configurations predicted by the present model seem to have more branches, in accordance with experimental observations [see Billings (1966)]. Up to this point, the sample calculations were concerned with individual dendrite growth. The expected simultaneous growth of den-

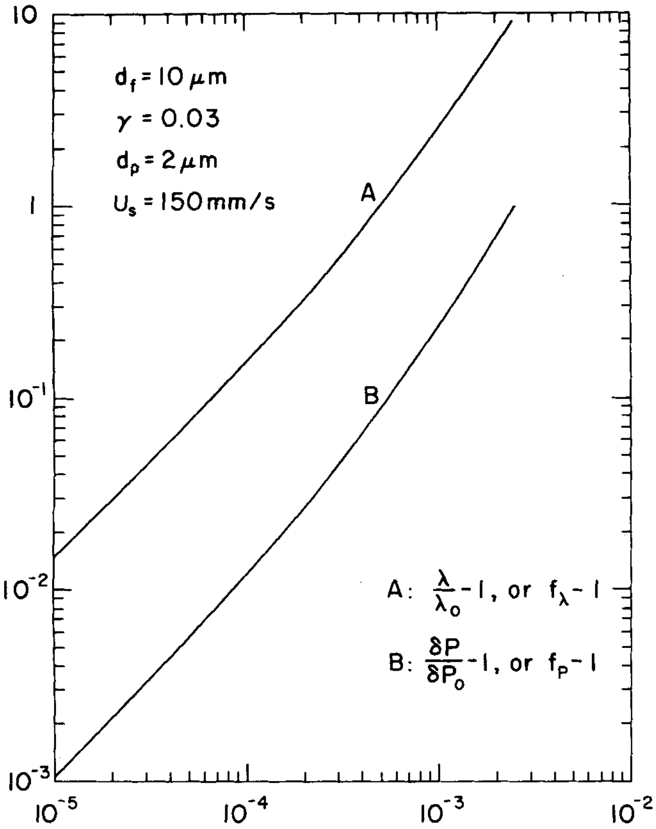

Fig. 8. Dependence of filter coefficient and pressure gradient on volume fraction of deposited matter.

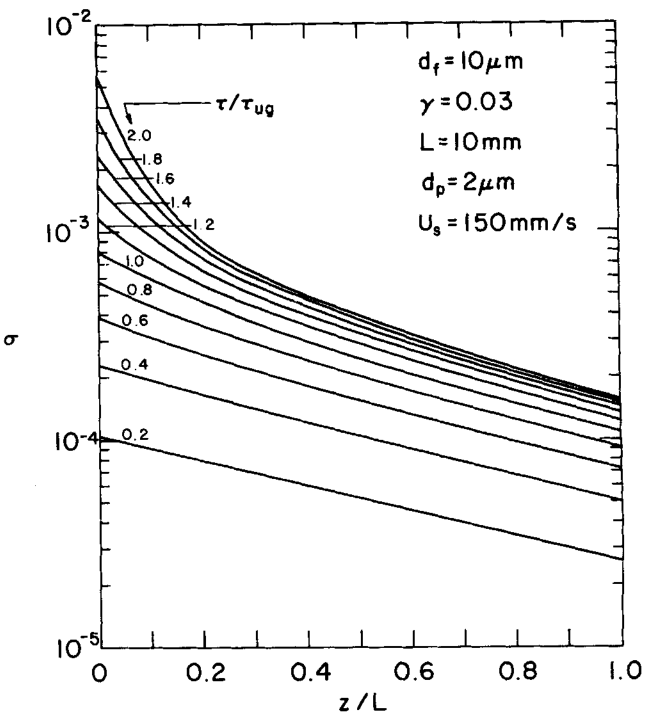

Fig. 9. Dependence of the volume fraction of deposited matter σ on normalized depth z/L and normalized time τ/τ_{ug} for the parameter values in Table 1.

drites on a given fiber is also predicted by the model of this work. For the conditions in Table 1, a typical dendrite growth on a fiber of length $l = 100\ \mu m$ at time $\tau = 1.5\ \tau_{ug}$ corresponding to the expected one is shown in Figure 6. As can be seen, the number of dendrites of any size (or age) per unit area increases rapidly with angle in the range $\pi/2 \leq \theta \leq \pi$. The largest dendrites, however, are located in the range $2\pi/3 \leq \theta \leq 8\pi/9$. The resemblance with the dendritic deposition observed experimentally by Billings (1966) is striking. It is also clear that even at this advanced stage, $\tau = 1.5\ \tau_{ug}$, dendrite-dendrite interference is still not a dominant factor, although its effect is not quite negligible either. Thus, it seems that the estimate of τ_{ug} given by Equation (32) is reasonable, and that in the present case one can use the proposed model up to, say, $\tau = 2\ \tau_{ug}$ without excessive error.

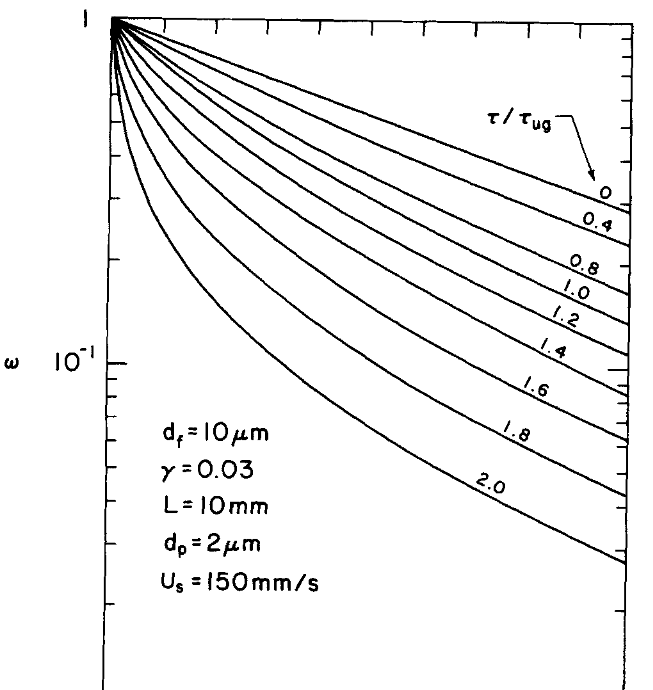

Fig. 10. Dependence of the normalized aerosol concentration ω on normalized depth z/L and normalized time τ/τ_{ug} for the parameter values in Table 1.

The dependence of the filter coefficient λ, of the pressure drop across a layer of differential thickness δP, and of the volume fraction of deposited matter σ on normalized time τ/τ_{ug} is shown in Figure 7, also for the values in Table 1. These results can be used to plot λ/λ_o and $\delta P/\delta P_o$ vs. σ by eliminating time τ as the common parameter, Figure 8. Figure 8 shows clearly the dramatic effect that even small deposited amounts have on the filter coefficient and the pressure drop. For instance, for $\sigma = 2 \times 10^{-3}$ we have $\lambda \cong 7.8\lambda_o$ and $\delta P \cong 1.64\ \delta P_o$, or a 680 and a 64% increase, respectively.

According to Equations (42) and (43), the plots of $(\lambda/\lambda_o - 1)$ vs. σ and of $(\delta P/\delta P_o - 1)$ vs. σ in Figure 8 are also plots of $(f_\lambda - 1)$ vs. σ and of $(f_P - 1)$ vs. σ, respectively. Using polynomial regression, we readily obtain the following two expressions:

$$f_\lambda = 1 + 0.14456 \times 10^4\sigma + 0.11153 \times 10^7\sigma^2 - 0.87710 \times 10^8\sigma^3 \quad (50)$$

$$f_P = 1 + 0.10720 \times 10^3\sigma + 0.11632 \times 10^6\sigma^2 - 0.20657 \times 10^7\sigma^3 \quad (51)$$

The coefficients on the right-hand sides of the above equations are not adjustable parameters, since Equation (50) is obtained by eliminating $\alpha\tau$ between Equations (35) and (37), and Equation (51) is obtained by eliminating $\alpha\tau$ between Equations (36) and (37) using third-

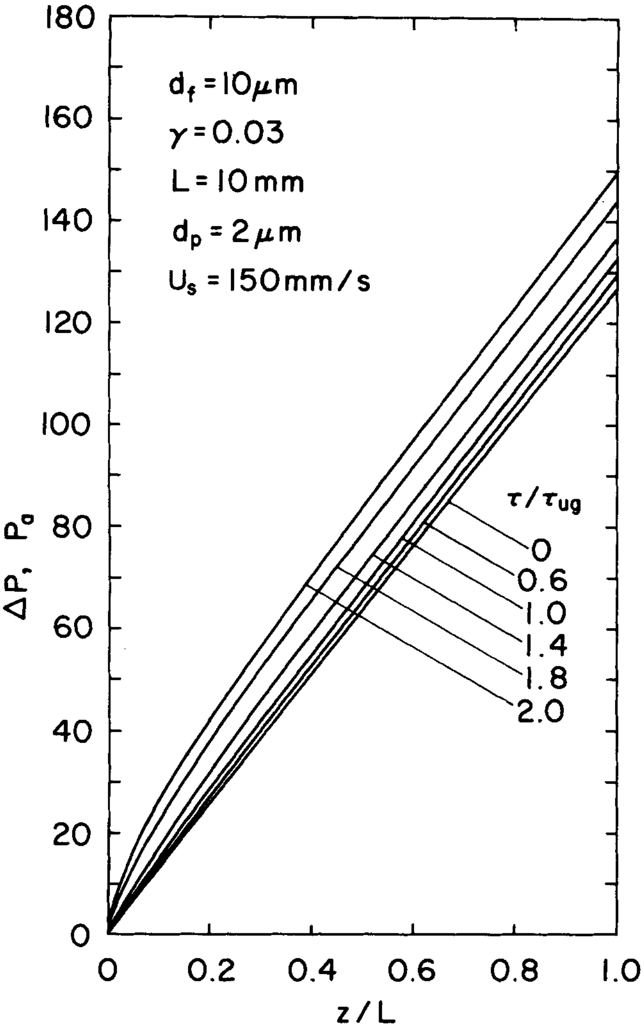

Fig. 11. Dependence of the pressure drop ΔP on normalized depth z/L and normalized time τ/τ_{ug} for the parameter values in Table 1.

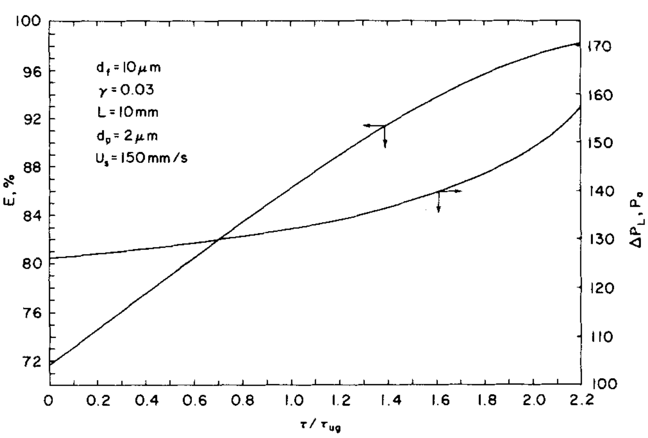

Fig. 12. Dependence of filter efficiency E and pressure drop across the filter ΔP_L on normalized time τ/τ_{ug} for the parameter values in Table 1.

degree polynomial regression. Thus, Equations (50) and (51) are analytical approximations to the formal Equations (42) and (43), respectively, and they can be used to predict the transient behavior of the corresponding filter of finite thickness L. Letting $L = 10$ mm and substituting the above expressions for f_λ and f_P in the system of differential equations developed in Payatakes (1976*b*), one obtains upon integration the volume fraction of deposited matter as a function of depth and time $\sigma(z, \tau)$, the normalized aerosol concentration as a function of depth and time $\omega(z, \tau)$, the pressure drop as a function of depth and time $\Delta P(z, \tau)$, and the filter efficiency as a function of time $E(\tau)$. The obtained results are summarized in Figures 9 through 12.

The filter efficiency E and total pressure drop ΔP_L for this example are plotted against normalized time τ/τ_{ug} in Figure 12. The changes observed are profound. During the relatively short initial period $0 \leq \tau \leq 2\tau_{ug}$, the filter efficiency increases from an initial value of 71.76% to a value of 97.23% at the expense of an increase of the total pressure drop from an initial value of 126.4 Pa (1.29 cm water gauge) to a value of 149.5 Pa (1.53 cm water gauge). Thus, it becomes clear that the modeling of the transient behavior is a task of the highest importance for the analysis and design of fibrous filtration systems.

A comment pertaining to the accuracy of the above sample calculations must be made here. These calculations were based on the assumption that the only deposition mechanism was direct interception. This, of course, is not true. For the conditions in Table 1, inertial impaction is also expected to be significant. A measure of the magnitude of the error committed by neglecting inertial effects is obtained by comparing the values of the single clean fiber collection efficiency η_o that are calculated with and without inertial effects. Based on the Kuwabara flow field, Stechkina et al. (1969) developed expressions for the calculation of the clean fiber collection efficiency due to various deposition mechanisms. For the conditions in Table 1, the aforementioned method predicts that 39% of the collection efficiency is due to interception, with 59% contributed by inertia and about 2% by Brownian diffusion. Obviously, the error committed by neglecting inertial effects is serious. This underlines the need for extension of the present work to include other deposition mechanisms. To this end the framework of the present work can be used intact. The only changes will involve the calculation of $\phi^{(\theta)}_{k-1,k}$, $\phi^{(\theta)}_{k,k}$, $\phi^{(r)}_{k-1,k}$, $\phi^{(r)}_{k,k}$ and $\nu(\theta)$. This is part of planned future work.

ACKNOWLEDGMENT

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NOTATION

a_f = fiber radius
$a_k(\theta)$ = functions defined by Equation (8)
a_p = particle radius
$b = \dfrac{a_f}{\sqrt{\gamma}}$ = radius of outer boundary of the cylindrical Kuwabara cell
$b_k(\theta)$ = functions defined by Equation (9)
$C_{ki}(\theta)$ = functions defined by Equation (12)
C_s = Cunningham slip factor, given by Equation (24)
d_f = fiber diameter
d_p = particle diameter
$E(\tau)$ = filter efficiency at time τ
$F(\alpha\tau)$ = function defined by Equation (40)
F_{pk} = drag force acting on a particle in the k^{th} layer of the dendrite
$f_k{}^\theta$, $f_k{}^r$ = drag force correction factors for tangential and normal flow, respectively

f_P = function expressing the effect of the amount of deposited matter on the local pressure gradient, defined by Equation (39)
f_λ = function expressing the effect of the amount of deposited matter on the local filter coefficient, defined by Equation (38)
i, j, k = indexes
$\mathbf{i}_r, \mathbf{i}_\theta$ = unit vectors of the cylindrical system or coordinates (r, θ), Figure 1
K = Kuwabara's constant, Equation (22)
L = filter thickness (finite)
$M(\theta)$ = integer function of θ defined by Equation (19)
M_k = actual number of particles in the k^{th} layer of a dendrite
m_k = expected particle number in the k^{th} layer of a dendrite
$N(\tau, \theta)$ = frequency function of number of dendrites per unit length of fiber at time τ and angle θ, taking into account both sides of the fiber
n = number of particles per unit aerosol volume
P = static pressure
r = radial cylindrical coordinate, Figure 1
$R_{i,j}^{(s)}$ = rate of increase of m_j by deposition on particles occupying the i^{th} layer due to the flow component in the s direction ($s = \theta$, or r)
$\mathcal{R}_{k-1,k}, \mathcal{R}_{k,k}$ = constants, defined by Equations (49)
t = time measured from the instant of deposition of the first particle of the dendrite, (dendrite age)
$U = \dfrac{U_s}{(1-\gamma)}$ = velocity of approach to fiber; also, mean interstitial velocity through the filter
U_s = superficial velocity through the filter
v_r, v_θ = radial and angular velocities in the Kuwabara unit cell, Equations (25) and (26), respectively
$W_{rk}, W_{\theta k}$ = constants, defined by Equations (28) and (29), respectively
X_k = constants, defined by Equation (30)
x = Cartesian coordinate, normal to main direction of flow, Figure 1
z = Cartesian coordinate in the main direction of flow, Figure 1

Greek Letters

α = rate of particles approaching a clean fiber per unit length, Equation (6)
$\boldsymbol{\beta}$ = parameter vector on which f_λ depends
γ = packing density of fibrous medium
$\Delta P(z, \tau)$ = pressure drop up to depth z at time τ
$\Delta P_L(\tau)$ = pressure drop across filter of thickness L at time τ
$\boldsymbol{\delta}$ = parameter vector on which f_P depends
$\delta P, \delta P_o$ = pressure drop and initial pressure drop across a fibrous filter with differential thickness δz
ϵ = small positive number (≤ 0.5), Equation (19)
η_o = single clean fiber collision efficiency (the adhesion efficiency is assumed $\eta_A = 1$)
$\Theta_{k-1,k}, \Theta_{k,k}$ = constants, defined by Equations (48)
θ = angular cylindrical coordinate, measured counterclockwise from the downstream stagnation point, Figure 1
λ = filter coefficient, defined by Equation (15)
λ_o = value of λ for a clean filter
λ' = mean free path of gas
μ = dynamic viscosity
$\nu(\theta)$ = function of θ defined by Equation (34)
ρ = coordination number, that is, maximum number of particles in the $(k+1)$st layer which can be attached directly to the same particle of the k^{th} layer
σ = specific deposit (volume fraction of deposited matter)
τ = time measured from the beginning of aerosol flow through the filter
τ_f = duration of filtration run
τ_{ug} = duration of period of unhindered dendrite growth, Equation (32)
$\Phi(\sigma)$ = formal function defined by Equation (41)
$\phi_{i,j}^{(s)}$ = functions defined so that $\alpha\phi_{i,j}^{(s)}$ is the rate of increase of m_j by deposition on a particle in the i^{th} layer due to the flow component in the s direction ($s = r$, or θ), assuming that deposition is unhindered by other particles
$\chi(t; \tau, \theta)$ = dendrite age distribution function at time τ and angular position θ
$\omega(z, \tau)$ = aerosol concentration at depth z and time τ normalized by the value of the feed concentration at time τ

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